Affine Transformations

- Arbitrary Rotations:
  - Somewhat complex, and no reason to know how to derive it
  - For a rotation of ϴ around the vector v, the 3×3 rotation matrix is:

$$R_{matr} = \vec{u}\vec{u}^T + (\cos\theta)\left(I_{matr} - \vec{u}\vec{u}^T\right) + (\sin\theta)S_{matr}$$

  - Where u, S_{matr}, and I_{matr} are:

$$\vec{u} = \begin{pmatrix} u_x \\ u_y \\ u_z \end{pmatrix} = \begin{pmatrix} v_x / \|\vec{v}\| \\ v_y / \|\vec{v}\| \\ v_z / \|\vec{v}\| \end{pmatrix}$$

$$S_{matr} = \begin{bmatrix} 0 & -u_z & u_y \\ u_z & 0 & -u_x \\ -u_y & u_x & 0 \end{bmatrix} \quad I_{matr} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \vec{u}\vec{u}^T = \begin{bmatrix} u_x u_x & u_x u_y & u_x u_z \\ u_y u_x & u_y u_y & u_y u_z \\ u_z u_x & u_z u_y & u_z u_z \end{bmatrix}$$

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Arbitrary Rotations

- Example:
  - What is the matrix that rotates by 45° around the axis (2, 1, 2) ?

$$R_{matr} = \vec{u}\vec{u}^T + (\cos\theta)\left(I_{matr} - \vec{u}\vec{u}^T\right) + (\sin\theta)S_{matr}$$

$$S_{matr} = \begin{bmatrix} 0 & -u_z & u_y \\ u_z & 0 & -u_x \\ -u_y & u_x & 0 \end{bmatrix} \quad I_{matr} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \vec{u}\vec{u}^T = \begin{bmatrix} u_x u_x & u_x u_y & u_x u_z \\ u_y u_x & u_y u_y & u_y u_z \\ u_z u_x & u_z u_y & u_z u_z \end{bmatrix} \quad \vec{u} = \begin{pmatrix} u_x \\ u_y \\ u_z \end{pmatrix} = \begin{pmatrix} v_x/\|\vec{v}\| \\ v_y/\|\vec{v}\| \\ v_z/\|\vec{v}\| \end{pmatrix}$$